6. M. Hason and A. S. Mujumdar, Coupled heat and mass transfer in natural convection under flux condition along a vertical cone, *Int. Commun. Heat Mass Transfer* **11**, 157–172 (1984).
7. T. S. Lee, R. G. Parikh, A. Acrivos and D. Bershader, Natural convection in a vertical channel with opposing buoyancy forces, *Int. J. Heat Mass Transfer* **25**, 499–511 (1982).
8. C. J. Chang, T. F. Lin and W. M. Yan, Natural convection flows in a vertical open tube resulting from combined buoyancy effects of thermal and mass diffusion, *Int. J. Heat Mass Transfer* **29**, 1543–1552 (1986).
9. W. M. Yan, Buoyancy induced heat and mass transfer in vertical channel flows, Ph.D. Thesis, Department of Mechanical Engineering, National Chiao Tung University, Hsinchu, Taiwan, January (1989).
10. W. M. Yan and T. F. Lin, Combined heat and mass transfer in natural convection between vertical parallel plates with film evaporation, *Int. J. Heat Mass Transfer* **33**, 529–541 (1990).
11. L. C. Chow and J. N. Chung, Evaporation of water into a laminar stream of air and superheated steam, *Int. J. Heat Mass Transfer* **26**, 373–380 (1983).
12. C. H. Wu, D. C. Davis, J. N. Chung and L. C. Chow, Simulation of wedge-shaped product dehydration using mixtures of superheated steam and air in laminar flow, *Numer. Heat Transfer* **11**, 109–123 (1987).
13. M. Haji and L. C. Chow, Experimental measurement of water evaporation rates into air and superheated steam, *J. Heat Transfer* **110**, 237–242 (1988).
14. T. F. Lin, C. J. Chang and W. M. Yan, Analysis of combined buoyancy effects of thermal and mass diffusion on laminar forced convection heat transfer in a vertical tube, *J. Heat Transfer* **110**, 337–344 (1988).
15. W. M. Yan, Y. L. Tsay and T. F. Lin, Simultaneous heat and mass transfer in laminar mixed convection flows between vertical parallel plates with asymmetric heating, *Int. J. Heat Fluid Flow* **10**, 262–269 (1989).
16. E. R. G. Eckert and R. M. Drake, Jr., *Analysis of Heat and Mass Transfer*, Chaps 20 and 22. McGraw-Hill, New York (1972).
17. J. L. Mangararo and O. T. Hanna, Simultaneous energy and mass transfer in the laminar boundary layer with large mass transfer rates toward the surface, *A.I.Ch.E. Jl* **16**, 204–211 (1970).
18. G. L. Hubbard, V. E. Denny and A. E. Mills, Droplet evaporation: effects of transients and variable properties, *Int. J. Heat Mass Transfer* **18**, 1003–1008 (1975).

Int. J. Heat Mass Transfer. Vol. 34, No. 4/5, pp. 1313–1316, 1991
Printed in Great Britain
0017-9310/91 $3.00+0.00

On the analysis of counter-flow cooling towers

FLETCHER OSTERLE
Department of Mechanical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, U.S.A.

(*Received* 11 *January* 1990 *and in final form* 4 *June* 1990)

INTRODUCTION

UNTIL recently the performance of counter-flow cooling towers was most commonly analyzed by applying the so-called Merkel model. This model is based on equations developed in a paper published in German by Merkel in 1925 [1]. This work was largely neglected until the paper was translated into English by Nottage in 1941 [2]. Since then the model has been widely applied. The equations express an energy balance and describe simultaneous mass and heat transfer coupled through the Lewis relation. However, in the interest of tractability the equations were simplified by omitting a term and as a result do not account for the mass of water lost by evaporation. Given the inlet water and air conditions the Merkel equations predict the enthalpy (hence wet-bulb temperature) of the outlet air, but not its humidity. The equations also predict the required number of transfer units (*NTU*) to accomplish the process. The *NTU* expresses the relationship which must exist between the mass/heat transfer coefficient and the tower volume to make the process possible. The Merkel equations are readily solved numerically using the modified Euler procedure, which can easily be executed in a spreadsheet format on a PC.

Recently a computer program called VERA2D [3] was developed for the Electric Power Research Institute (EPRI) to provide a two-dimensional model for determining cooling tower performance, both counter-flow and cross-flow. VERA2D also corrects an error in the derivation of the Merkel equations. The present paper retains the assumption of a one-dimensional model but corrects the Merkel equations so that the mass of water lost by evaporation is properly accounted for. Consequently with the model developed here the enthalpy and humidity of the air exiting the tower are determined. Corrected values of *NTU* are also evaluated. Now, a set of differential equations must be solved rather than just one. However, again using the modified Euler procedure the solution is readily executed in a spreadsheet format on a PC. It is found that the Merkel equations underestimate the required *NTU* by an amount which can be significant. In addition to improving the prediction of the required *NTU*, this model predicts the state of the outlet air, not just its enthalpy. It is necessary to know the state of this air if the effect on the environment of the cooling tower operation is to be determined; by, for example, entropy or exergy calculations.

BASIC THEORY

Consider a vertical counter-flow cooling tower in which liquid water enters the top at a mass flow rate L_1 and a temperature t_1, and leaves the bottom at a mass flow rate L_2 (less than L_1 due to evaporation) and a temperature t_2. Air enters the bottom of the tower at a mass flow rate $G(1+X_3)$ (where G is the mass flow rate of dry air and X is the absolute humidity; i.e. mass of water vapor per unit mass of dry air), a dry-bulb temperature T_3 and a wet-bulb teperature T_3^*, and leaves at the top at a temperature T_4 and an absolute humidity X_4.

L_1, t_1, t_2, G, T_3, and T_3^* (hence X_3) will be considered given as describing a particular cooling task; i.e. L_1 cooled from t_1 to t_2 (with some evaporation) by a dry air flow G with atmospheric conditions described by T_3 and T_3^*. The problem is to determine T_4, X_4 (and hence the liquid water loss due to evaporation), and the relationship between the mass and heat transfer coefficients and the tower volume required to perform the operation. This will be accomplished by requiring mass and energy conservation, and applying appropriate mass and heat transfer relationships.

We will adopt a one-dimensional model by assuming that the state of the water and air varies only with vertical position

NOMENCLATURE

a	air/water interface area per unit tower volume [$m^2 m^{-3}$]
C_{pa}	specific heat of dry air [$kJ\,kg^{-1}\,^\circ C^{-1}$]
C_{pl}	specific heat of liquid water [$kJ\,kg^{-1}\,^\circ C^{-1}$]
C_{pv}	specific heat of water vapor [$kJ\,kg^{-1}\,^\circ C^{-1}$]
F	dimensionless quantity defined by equation (21)
G	dry air flow rate [$kg\,s^{-1}$]
h	enthalpy of moist air at temperature T [$kJ\,kg^{-1}$]
h_c	mass transfer coefficient [$m\,s^{-1}$]
h_t	heat transfer coefficient [$W\,m^{-2}\,^\circ C^{-1}$]
H	enthalpy of saturated water vapor at temperature t [$kJ\,kg^{-1}$]
H_0	latent heat of water at reference temperature T_0 [$kJ\,kg^{-1}$]
K	$h_c\rho$ [$kg\,m^{-2}\,s^{-1}$]
L	liquid water flow [$kg\,s^{-1}$]
NTU	number of heat transfer units, KaV/L_1 [dimensionless]
q	total heat transfer rate [W]
q_s	sensible heat transfer rate [W]
t	liquid water temperature [$^\circ C$]
T	moist air temperature at level where liquid water temperature is t [$^\circ C$]
T_0	reference temperature for enthalpy of air and water [$^\circ C$]
V	tower volume [m^3]
X	absolute humidity of moist air [kg water (kg dry air)$^{-1}$].

Greek symbol

ρ	density of dry air.

Subscripts

1	state of liquid water entering top of tower
2	state of liquid water leaving bottom of tower
3	state of air entering bottom of tower
4	state of air leaving top of tower.

Superscript

*	saturated at temperature t.

in the tower. Considering a differential section of the tower of volume dV, conservation of mass requires that

$$dL = G\,dX \tag{1}$$

where L is the liquid water flow (down) and X the humidity of the air at the level in the tower where the water temperature is t (measured above some reference level). Assuming adiabatic flow, conservation of energy requires that

$$C_{pl}\,d(tL) = G\,dh \tag{2}$$

where C_{pl} is the specific heat of the liquid water, h the enthalpy of the humid air in the tower at the level where the liquid water temperature is t, and G the dry air flow (up). The enthalpy of humid air at temperature, T, is given by

$$h = C_{pa}(T-T_0)+X[C_{pv}(T-T_0)+H_0] \tag{3}$$

where C_{pa} is the specific heat of dry air, C_{pv} the specific heat of water vapor, T_0 the reference temperature for the enthalpy of moist air (0°C for the Goff–Gratch tables [4]), and H_0 the latent heat at the reference temperature. For consistency, t will now be in °C. Combining equations (1) and (2), we have

$$G\,dh = C_{pl}(L\,dt+tG\,dX). \tag{4}$$

The Merkel equations neglect the second term on the right-hand side of equation (4), which, by equation (1), makes L a constant. Thus, evaporation is neglected in the Merkel equations.

To find the differential tower volume, dV, in which the temperature change, dt, occurs, it is necessary to consider mass and heat transfer rates between the liquid water and the humid air at the level in the tower where the temperature is t. It will be assumed that the air in contact with the liquid water is saturated at the water temperature. In the differential volume, dV, the sensible heat transfer is given by

$$dq_s = h_t a\,dV(t-T) \tag{5}$$

where h_t is the heat transfer coefficient, and a the transfer surface area per unit tower volume. T is the temperature of the humid air 'engaging' the water at this level. The total heat transfer (including latent heat) is given by

$$dq = dq_s + H\,dL \tag{6}$$

where dL is the amount of water evaporated in dV, and H the enthalpy of saturated vapor at temperature t. The Merkel analysis uses the latent heat instead of the vapor enthalpy in this equation. VERA2D corrects this. The mass (water) transfer in the differential tower volume, dV, is given by

$$dL = h_c\rho a\,dV(X^*-X) \tag{7}$$

where h_c is the mass transfer coefficient, ρ the air density, and X^* the absolute humidity of saturated air at t. Now for the air–water vapor system

$$\frac{h_t}{\rho C_{pg} h_c} \cong 1 \tag{8}$$

the Lewis relation, where C_{pg} is the specific heat of humid air. Substituting from equations (5), (7), and (8), equation (6) becomes

$$dq = h_c\rho a\,dV[C_{pg}(t-T)+H(X^*-X)]. \tag{9}$$

Equation (3) can be regrouped to read

$$h = (C_{pa}+XC_{pv})T+X(H_0-C_{pv}T_0)-C_{pa}T_0 \tag{10}$$

where the term multiplying T is C_{pg}, the specific heat of humid air, which is nearly constant and differs little from C_{pa}.

Writing equation (10) for saturated air at the level where the water temperature is t, and subtracting equation (10) from this equation there results

$$h^*-h = C_{pg}(t-T)+(H_0-C_{pv}T_0)(X^*-X). \tag{11}$$

Now substituting equation (11) into equation (9), there results

$$dq = h_c\rho a\,dV[(h^*-h)+\{(H-H_0)+C_{pv}T_0\}(X^*-X)]. \tag{12}$$

It turns out that the second term in square brackets is small (contributing of the order of 2%) and will be neglected. Since this term is neglected, the use of latent heat rather than vapor enthalpy in equation (6) is of no consequence in Merkel's (or our) results. Equation (12) becomes

$$dq = h_c\rho a\,dV(h^*-h). \tag{13}$$

Now

$$dq = G\,dh. \tag{14}$$

Substituting equations (13) and (4) in equation (14) yields

$$Ka\,dV(h^*-h) = C_{pl}(L\,dt+tG\,dX) \tag{15}$$

and from equation (1) into equation (7)

$$Ka\,dV(X^*-X) = G\,dX. \tag{16}$$

Here K has been written for $h_c\rho$ as is done in the literature.

The basic equations governing counter-flow cooling tower performance have now been established. They are equations (1), (4), (15), and (16), and will now be rearranged into a form suitable for solution.

By eliminating $Ka\,dV$ between equations (15) and (16) we obtain an equation for X, the humidity. Inserting this equation into equation (4) we obtain an equation for h, the enthalpy. Finally, inserting these expressions into equation (16) we obtain an equation for $Ka\,dV$. After all of these manipulations there results the following set of governing equations:

$$G\,dh = C_{pl}L\,dt\left[\frac{1}{1-F}\right] \tag{17}$$

$$Gt\,dX = L\,dt\left[\frac{F}{1-F}\right] \tag{18}$$

$$d(KaV) = \frac{LC_{pl}\,dt}{h^*-h}\left[\frac{1}{1-F}\right] \tag{19}$$

where

$$F = \frac{tC_{pl}[X^*-X]}{h^*-h} \tag{20}$$

and from equation (1)

$$dL = G\,dX \tag{21}$$

from which

$$L = L_2 + G[X - X_3]. \tag{22}$$

The variable F which appears in these equations has a magnitude of the order of 0.1. If it is neglected compared to unity, the set of equations reduce to the Merkel equations, which greatly simplify the analysis because now the enthalpy is immediately established at all values of the temperature by the integration of equation (17) and KaV readily established by the numerical integration of equation (19) (with h^* values obtained from the Goff–Gratch tables).

CORROBORATION OF EQUATIONS

Baker [5] gives the history of the development of the cooling tower equations, including an extensive list of references. More references are to be found in refs. [3, 6]. A comparison of our model with Merkel's has just been made. In his book Baker presents a rigorous derivation of the cooling tower governing equations which he compares with the Merkel equations. The matter of the use of latent heat rather than saturated vapor enthalpy has already been pointed out. Of the model governing equations derived here; namely, equations (1), (4), (15), and (16), equations (1) and (16) agree with Baker's derivation. Equation (4) should agree with his equation (6.6) but differs in that he places a t_2 where we have a t. This is the result of a mistake made in going from a difference representation to a differential representation. This occurs in the term which the Merkel model neglects but we do not. Equation (15) should agree with Baker's equation (6.15) and does after a typographical error is corrected.

EXAMPLE PROBLEM

Consider the following cooling tower design specifications:

liquid water enters the top of the tower at $t_1 = 43.3$°C (110°F);
liquid water leaves the bottom of the tower at $t_2 = 28.9$°C (84°F);
humid air enters the bottom of the tower at a wet-bulb temperature $T_3^* = 20.6$ °C (69°F);
liquid water to air flow ratio, L_1/G, is 1.3.

Table 1. Tower exit conditions

X_3	h_4 (kJ kg^{-1})	X_4	KaV/L_1
0.015	141.1	0.0409	1.654
0.010	141.7	0.0405	1.681
0.005	142.3	0.0401	1.708

For the Merkel model (which neglects evaporation) this is all that needs to be specified. The model predicts the enthalpy of the exiting air, h_4, and the NTU, KaV/L_1. However, for the model presented here (which accounts for evaporation) the humidity of the entering air, X_3 (or, alternatively, the dry-bulb temperature), must be specified, and the model predicts the humidity, X_4, as well as the enthalpy of the exiting air, along with NTU. Air at a wet-bulb temperature of 20.6°C is saturated at a humidity of 0.015 kg water (kg dry air)$^{-1}$. We consider three values of X_4: 0.015, 0.010, and 0.005, representing progressively drier air. In all three of these cases the Merkel model will give the same results for h_4 and KaV/L_1.

The Merkel solution was obtained by the modified Euler integration scheme using a 1/1.8°C (1°F) interval in liquid water temperature. The results are

$$h_4 = 138.1\ \text{kJ kg}^{-1}$$

$$\frac{KaV}{L_1} = 1.514$$

$$T_4^* = 36°\text{C}.$$

T_4 and X_4 are not determined.

Now the model developed in this paper will be applied to this example. The governing equations are equations (17)–(19), with L given by equation (21) and F defined by equation (20). We have three simultaneous non-linear differential equations expressing h, X, and KaV/L_1 as functions of h, X, and the water temperature t. They were solved numerically by the modified Euler scheme in a spreadsheet format with a 1/1.8°C temperature interval to give the results shown in Table 1. In all three cases the air exiting the tower is very nearly saturated at 37°C, as expected. Comparing these results with those obtained from the Merkel model we see that the most substantial difference is in the prediction of KaV/L_1, which is significantly lower by the Merkel model.

CONCLUSIONS

The cooling tower model developed in this paper corrects the Merkel equations by including a term in the energy balance which the Merkel equations neglect, thus enabling the state of the exiting air to be determined and providing a more accurate prediction of the required NTU. The numerical analysis is easily carried out in a spreadsheet format on a PC. The required NTU as predicted by this model can be significantly different from that predicted by the Merkel model.

REFERENCES

1. F. Merkel, Verdunstungskuehlung, *VDI Forschungarbeiten* No. 275 (1925).
2. H. B. Nottage, Merkel's cooling diagram as a performance correlation for air–water evaporative cooling systems, *ASHVE Trans.* **47**, 429 (1941).
3. A. K. Majumdar, A. K. Singhal and D. B. Spalding, VERA2D: program for 2-D analysis of flow, heat, and mass transfer in evaporative cooling towers, Vol. 1: mathematical formulation, solution procedure, and applications, EPRI CS-2923, Project 1262-1, Final Report (March 1983). Also: Numerical modeling of wet cooling

towers: Parts I and II, *ASME J. Heat Transfer* **105**, 728 743 (1983).
4. J. A. Goff and S. Gratch, Thermodynamic properties of moist air. In *Cooling Tower Performance*, Appendix D. Chemical, New York (1984).
5. D. R. Baker, *Cooling Tower Performance*. Chemical, New York (1984).
6. N. W. Kelly, *Kelly's Handbook of Crossflow Tower Performance*. Neil W. Kelly and Associates, Kansas City, Missouri (1976).

Int. J. Heat Mass Transfer. Vol. 34, No. 4/5, pp. 1316 1318, 1991
Printed in Great Britain
0017 9310/91 $3.00+0.00
Pergamon Press plc

New method for data reduction in flash method

L. VOZÁR, J. GEMBAROVIČ and V. MAJERNÍK
Katedra fyziky, Pedagogická fakulta, Lomonosovova 1, Nitra CS-949 74, Czechoslovakia

(*Received* 30 *April* 1990)

INTRODUCTION

THE PULSE 'flash' method [1] is actually the most popular method of measuring the thermal diffusivity of solids, especially at high temperatures. In this method, the front face of a small disc-shaped specimen is subjected to a very short burst of radiant energy coming from either a laser or a xenon flash lamp. The resulting temperature rise of the rear surface of the specimen is recorded and the value of the thermal diffusivity is computed from this temperature vs time data.

There exist several original methods for data reduction in the flash method. The first group includes a method in which the thermal diffusivity is calculated using one or a few characteristic experimental points [1, 2]. In the second group all the experimental points of temperature vs time data (in particular their rising part) are used for diffusivity determination. Such methods are based on fitting the experimental data by theoretical curve by means of a least square procedure [3–5]. In all the mentioned methods the precision of results depends on satisfying the conditions, which are assumed in the ideal theoretical model of the flash method [1] (heat pulse is uniform and instantaneous; sample is opaque, homogeneous, and thermally insulated; thermal properties are temperature independent).

In the real case heat transfer between the sample and its environment is usually unavoidable, especially for high temperature measurements or for materials with poor conductivity. Several original methods were proposed which take into account this effect. In refs. [6–9] the thermal diffusivity determined from the ideal condition model is corrected by multiplying with the appropriate numerical factor, depending on heat losses. Another way is used in methods based on the general mathematical model [10] obtained as a solution to a two-dimensional heat conduction equation with the heat losses from the whole sample surface. In those methods the thermal diffusivity is determined either by means of several particular points of the temperature vs time data [10, 11] or using the temporal moments of the defined temperature interval of the rising part of the experimental curve [10, 12]. An original way to eliminate the heat loss effect, described in ref. [13], is based on the knowledge, that rear surface temperature history is less perturbed as time is nearer to the time origin (time of flash). Thermal diffusivity is obtained by extrapolating the time evolution of experimentally gained values of 'apparent' thermal diffusivity to time zero.

In this paper the data reduction method is presented, which eliminates the effect of heat losses using the procedure of extrapolation, similar to that in ref. [13]. Apparent values of thermal diffusivity are calculated using the so-called 'logarithmic' method [5]. Results of testing and comparison with other existing methods are also given.

PRINCIPLE OF METHOD

The logarithmic method [5] for thermal diffusivity determination is based on the relation

$$\ln(t^{1/2}T) = \ln[2T_{\text{lim}}(e^2/\pi\alpha)^{1/2}] - e^2/4\alpha t \quad (1)$$

where t is time, $T = T(e, t)$ the temperature of the rear face of the sample, e the sample thickness, α the thermal diffusivity, and T_{lim} the adiabatic limit temperature of the sample after the pulse. Equation (1) is an approximation of the formula, derived from the one-dimensional heat conduction equation using Laplace transformation and can be used over the time region in which the condition $T/T_{\text{lim}} \leqslant 0.9$ is fulfilled.

In the ideal case the plot $\ln(t^{1/2}T)$ vs $1/t$ is a straight line, and the thermal diffusivity α is calculated by means of the slope K of this line using the formula

$$\alpha = -e^2/4K \quad (2)$$

which is independent of T_{lim}.

The adiabatic limit temperature T_{lim} can be calculated through the point of intersection Q of the regression line with the axis of ordinates. According to equation (1) we have

$$T_{\text{lim}} = (\pi\alpha)^{1/2}\exp(Q)/2e. \quad (3)$$

In the real case, when the heat transfer between the sample and its environment is non-zero, the experimental curve $\ln(t^{1/2}T)$ vs $1/t$ is distorted due to the effect of heat losses. Therefore, the slope K and the point of intersection Q of the regression line with the axis of ordinates became a function of the time point, around which the linear regression is used ($K = K(t)$ and $Q = Q(t)$). The apparent diffusivity $\alpha(t)$, and apparent limit temperature $T_{\text{lim}}(t)$ can be calculated using equations (2) and (3), respectively.

In the method presented the values of thermal diffusivity and adiabatic limit temperature are specified by extrapolating time evolutions of the apparent diffusivity and apparent limit temperature, respectively, towards the initial time. From this point of view, our method corresponds to the procedure described in ref. [13]. However, in our algorithm the apparent diffusivity is independent of the apparent limit temperature and, consequently, the thermal diffusivity is independent of the adiabatic limit temperature. In addition, our method enables one to determine the value of the adiabatic limit temperature when using the analogical procedure as in the case of thermal diffusivity.

We see that the procedure presented transforms the problem of correction to the effect of heat losses to a mathematical problem of regression analysis.

In order to find the value of thermal diffusivity from the time evolution of the apparent diffusivity the polynomial regression of second order is used

$$\alpha(t) = \alpha_0 + \alpha_2 t^2 \quad (4)$$